Multi-Factorial Risk Assessment: An Approach based on Fuzzy Preference Relations

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Abstract

The main purpose of this paper is to develop a new method to aggregate the information given by several experts or criteria about different alternatives in order to obtain the preferred alternative or alternatives. This method has to take into account the interaction of the different alternatives and a parameter modelling the flexibility of this method has to be introduced. More precisely, this method uses fuzzy preference relations, aggregated by means of weighted ordered weighted averaging aggregation operators (WOWA). For the exploitation phase the extended weighted voting algorithm is introduced and studied in detail. Finally, the goodness of this approach is analysed using it to combine different points of view (people, environment, assets and reputation impact for the company) in the assessment of risk associated with human reliability.

Key words: Fuzzy preference relation, Aggregation function, WOWA operator, Group decision making problem, Risk assessment matrix.

1 Introduction

Each year billions of dollars are spent to develop, manufacture, and operate transportation systems such as aircraft, ships, trains and motor vehicles throughout the world. During their operational life-time, thousands of lives are lost annually due to various types of accidents. For example, in the United States around 42000 deaths occur annually due to automobile accidents only on highways [16]. In terms of dollars, in 1994 the total cost of motor vehicle crashes, was estimated to be around $150 billion to the United States economy [11,16].

In addition, between the 70% and 90% of transportation crashes are produced as a consequence of human error to a certain degree [16]. Moreover, human errors contribute significantly to many transportation crashes across all modes of transportation. For example, according to a National Aeronautics and Space Administration (NASA) study over 70% of airline accidents involved some degree of human error and according to a British study around 70% of railway accidents on four main lines during the period 1900-1997 were the result of human error [1,12,13]. Although the study of human reliability may be traced back to 1958, but since lates 80s several hundreds of papers on human reliability have appeared. An interesting list can be found in the Appendix of the classical book of Dhillon [7].

A main topic related to human factors is the concept of risk assessment matrix (see [6,19]). It is considered useful for studying human reliability in general and human reliability in transportation systems in particular. The risk assessment matrix allows the classification of different kind of errors according to their importance. This classification can help in decision making about the most important or urgent one.

Usually the risk matrix takes into account only one criterion (most of the cases: economical impact). However, decision making in a company often considers more than one. Therefore, it is interesting to consider at the same time more than one different risk matrix, each one associated with a different criteria for consequences (for example, effects on people, environment, assets or reputation).

Thus, a method to combine this information is needed, in order to classify the errors according to more than one criterion. This is our starting point. However, we have developed a general method to combine the information about different alternatives given by several experts or taking into account several criteria and the choice of the set of the best ones. This method can be applied in any environment where exists interaction among the different alternatives or some experts are more reliable than others. This will be done by the definition of fuzzy preference relations and the use of different aggregation functions, in particular the weighted ordered weighted averaging operator.

Thus, firstly the general method is developed and later it is applied to the particular area we are interested in. In detail, the structure of this paper is as follows: Section 2 gives an overview of the preliminary definitions used in this article. In Section 3 the group decision making problem matter of this study and also the proposed method to solve it are presented and its behaviour in accordance to the used parameter is studied. Section 4 shows our experimental framework and the experimental analysis carried out for the particular case of human reliability. We finish with some conclusions and open problems.

2 Preliminary definitions

In this section we carry out a brief introduction to fuzzy preference relations and aggregation operators. Firstly, we will introduce the type of fuzzy relations used in this work, together with some specific properties. Next, we will recall the definitions of aggregation operators and, in particular, the case of the non-weighted and weighted ordered weighted averaging aggregation operators.

2.1 Fuzzy preference relations

Initially, in group decision making decision making problems crisp relations are used to represent the presence or absence of preference between the different alternatives. However, in real problems, it is hard to measure the preference between two alternatives and, in some cases, we cannot unequivocally determine which one is preferred. That is the reason why this concept is generalized by introducing multivalued relations. This generalization allows us to measure the degree of preference to which an alternative is preferred to another. A general study about multivalued relations can be seen in [9].

There are different kind of multivalued or fuzzy relations, according to different ways of considering the available information. In particular we are interested in probabilistic relations (also known as reciprocal or ipsodual relations). In this paper, as there is no ambiguity, we will call them fuzzy preference relation.

Definition 1 *Given a finite set of alternatives $\mathcal{A}$, a* ***fuzzy preference relation*** *P is a mapping $P : \mathcal{A} \times \mathcal{A} \to [0, 1]$ such that $P(a, b) + P(b, a) = 1$ for any pair of alternatives a and b in $\mathcal{A}$.*

The fuzzy preference relation $P(a, b)$ between two alternatives a and b carries a bipolar semantic, meaning that the interval $[0, 1/2)$ represents preference of

b over a, the interval $(1/2, 1]$ represents the preference of a over b and the central value $1/2$ represents indifference.

In group decision making problems fuzzy preference relations are often represented as a matrix:

$$P = \begin{pmatrix} p_{11} & \dots & p_{1n} \\ \vdots & \dots & \vdots \\ p_{n1} & \dots & p_{nn} \end{pmatrix},$$

where $p_{ij} = P(a_i, a_j)$ denotes the degree to which alternative a_i is preferred to alternative a_j.

When comparing fuzzy quantities, the absence of a global definition of transitivity could become a problem. In the literature (see [20]), several definitions of transitivity are available. Thus, for fuzzy preference relations, coherent properties are the following.

Definition 2 *Let P be a fuzzy preference relation on $\mathcal{A}$:*

- *P is said to be **weakly transitive** iff* $\forall (a, b, c) \in \mathcal{A}^3$*:*

$$P(a, b) > P(b, a) \text{ and } P(b, c) > P(c, b) \Rightarrow P(a, c) > P(c, a).$$

- *P is said to be **acyclic** iff* $\forall (a_1, a_2, \dots, a_n) \in \mathcal{A}^n$*:*

$$P(a_1, a_2) > P(a_2, a_1),\ P(a_2, a_3) > P(a_3, a_2), \dots,\ P(a_{n-1}, a_n) > P(a_n, a_{n-1})$$
$$\Rightarrow P(a_1, a_n) \geq P(a_n, a_1).$$

As it is logical, a coherent behaviour is necessary to order the alternatives, and this coherence could be established by any of the previous properties. A more detailed explanation about transitivity and its importance for ordering of fuzzy quantities can be seen in [21,22].

2.2 Aggregation functions

In a rather informal way, the aggregation problem consists in aggregating n-tuples of objects all belonging to a given set into a single object of the same set.

This is an indispensable tool in many fields as engineering, economical or social sciences. By this reason, the theoretical study of aggregation operators have increased a lot during the last years. A good overview about them can be found in [4]. In this section we will simply recall the most important definitions for our purposes.

Definition 3 *Let I be a closed interval in $\mathbb{R}$. An m-**ary aggregation operator or aggregation function** is a function*

$$\begin{array}{rccc} f: & I^m & \longrightarrow & I \\ & (x_1, x_2, \ldots, x_m) & \longrightarrow & f(x_1, x_2, \ldots, x_m) \end{array}$$

fulfilling the following properties:

- *Increasing:* $f(x_1, x_2, \ldots, x_m) \leq f(y_1, y_2, \ldots, y_m)$ *if* $x_i \leq y_i$ *for any* $i \in \{1, 2, \ldots, m\}$.
- *Boundary conditions:* $\inf\limits_{x \in I^m} f(x) = \inf I$ *and* $\sup\limits_{x \in I^m} f(x) = \sup I$.

In this paper, the particular case of ordered weighted averaging aggregation operators are considered. They were originally introduced by Yager in [23] to provide a method for aggregating scores associated with the satisfaction of multiple criteria.

Formally,

Definition 4 *Let I be a closed interval in $\mathbb{R}$. An **ordered weighted averaging aggregation operator** (OWA for short) is any aggregation operator defined by*

$$\begin{array}{rccc} f_{OWA}: & I^m & \longrightarrow & I \\ & (x_1, x_2, \ldots, x_m) & \longrightarrow & \sum\limits_{i=1}^{m} w_i x_{\sigma(i)} \end{array}$$

where σ is the permutation that sorts the elements in the following way: $x_{\sigma(1)} \geq x_{\sigma(2)} \geq \ldots \geq x_{\sigma(m)}$ and $\{w_i\}_{i=1}^{m}$ is a family of weights such that $w_i \geq 0$ and $\sum\limits_{i=1}^{m} w_i = 1$.

A complete study about these functions can be found in [24]. Examples of aggregation functions are the arithmetic or geometric means, minimum, maximum, median and the weighted arithmetic mean (WM). All of them, except the geometric mean and the weighted arithmetic mean, are examples of OWA functions.

The goal of a group decision making problem is to obtain, through consensus of the experts, the most accepted alternative or set of alternatives. The widespread idea of using OWA operators in this kind of problems is intuitive and logical, and it can be improved with the use of linguistic quantifiers as: "as many as possible" or "at least half" of the experts. However, sometimes there are experts more reliable than others, primarily when talking not about experts but different criteria or points of view. That is the reason why the WOWA operator is introduced ([18]).

Definition 5 *Let I be a closed interval in $\mathbb{R}$. A* ***weighted ordered weighted averaging aggregation operator*** *(WOWA for short) is any aggregation operator defined by*

$$\begin{array}{rccc} f_{WOWA}: & I^m & \longrightarrow & I \\ & (x_1, x_2, \ldots, x_m) & \longrightarrow & \displaystyle\sum_{i=1}^{m} \omega_i \, x_{\sigma(i)} \end{array}$$

where

- *$\{\sigma(1), \ldots, \sigma(m)\}$ is a permutation of $\{1, \ldots, m\}$ such that $x_{\sigma(i-1)} \geq x_{\sigma(i)}$ for all $i = 2, \ldots, m$ and*
- *the weight ω_i are defined for all $i = 2, \ldots, m$ as:*

$$\omega_i = w^* \left(\sum_{j \leq i} w^{WM}_{\sigma(j)} \right) - w^* \left(\sum_{j < i} w^{WM}_{\sigma(j)} \right),$$

with w^* a monotone increasing function that interpolates by means of straight lines the $m+1$ points $\left\{ (0,0), (1/m, w_1^{OWA}), \ldots, (i/m, \sum_{j \leq i} w_j^{OWA}), \ldots, (1,1) \right\}$, being w^{WM} and w^{OWA} the weighting vectors of dimension m, associated respectively with the weighted arithmetic mean and a OWA operator. That is, $w^{WM} = (w_1^{WM}, \ldots, w_n^{WM})$ and $w^{OWA} = (w_1^{OWA}, \ldots, w_n^{OWA})$ fulfilling that $w_i^{WM}, w_i^{OWA} \geq 0$ for all $i \in \{1, 2, \ldots, m\}$ and $\sum_i w_i^{WM} = \sum_i w_i^{OWA} = 1$.

The WOWA operator is an aggregation operator, i.e. it remains between the minimum and the maximum and it is increasing in all of its arguments. This operator can be seen as a generalization of weighted arithmetic mean and OWA operators.

Example 6 *Due to the definition of WOWA operator is not very simple, let us consider an example of application of them.*

Let $x = (1, 2, 3, 4)$ be the vector to be aggregated, w^* the linear interpolation and consider the weighting vectors $w^{WM} = (1/4, 1/4, 1/3, 1/6)$ and $w^{OWA} = (1/2, 1/3, 1/9, 1/18)$.

Firstly, the function w^* is defined. In this case, as the linear interpolation is considered, w^* is the polygonal line interpolating the points $(0,0)$, $(1/4, 1/2)$,

$(1/2, 5/6)$, $(3/4, 17/18)$ *and* $(1, 1)$. *Thus,*

$$w^*(x) = \begin{cases} 2\,x & \text{if } 0 \le x \le 1/4, \\ 4/3x + 1/6 & \text{if } 1/4 < x \le 1/2, \\ 4/9x + 11/18 & \text{if } 1/2 < x \le 3/4, \\ 2/9x + 7/9 & \text{if } 3/4 < x \le 1. \end{cases}$$

Now the weight ω *associated to vector* $x = (1, 2, 3, 4)$ *is computed. Thus,*

$$\begin{aligned} \omega_1 &= w^*(1/6) - w^*(0) = 1/3, \\ \omega_2 &= w^*(1/2) - w^*(1/6) = 1/2, \\ \omega_3 &= w^*(3/4) - w^*(1/2) = 1/9, \\ \omega_4 &= w^*(1) - w^*(3/4) = 1/18, \end{aligned}$$

and therefore, $\omega = (1/3, 1/2, 1/9, 1/18)$. *Finally, the aggregated value is obtained* $f_{WOWA}(1, 2, 3, 4) = \frac{1}{3} \cdot 4 + \frac{1}{2} \cdot 3 + \frac{1}{9} \cdot 2 + \frac{1}{18} \cdot 1 = 28/9$.

Let us notice that the weight vector ω depends on w^{WM} and w^{OWA}, but it also depends on the vector to be aggregated, $x = (x_1, \ldots, x_m)$ as we can see in the next example.

Example 7 *In Example 6, the vector to be aggregated was* $x = (1, 2, 3, 4)$, *which implied that* $w_\sigma^{WM} = (1/6, 1/3, 1/4, 1/4)$ *and from that, it was obtained that* $\omega = (1/3, 1/2, 1/9, 1/18)$.

However, if we want to aggregate the vector $(0.3810, 0.8889, 0.6154, 0.2353)$, *as the order of the elements is different, we have that* $w_\sigma^{WM} = (1/4, 1/3, 1/4, 1/6)$ *and therefore, the associated vector of weights is* $\omega = (1/2, 10/27, 5/54, 1/27)$.

In that case, $f_{WOWA}(0.3810, 0.8889, 0.6154, 0.2353) = \frac{1}{2} \cdot 0.8889 + \frac{10}{27} \cdot 0.6154 + \frac{5}{54} \cdot 0.3810 + \frac{1}{27} \cdot 0.2353 = 0.7164$.

In any case, for any vector x, the associated $\{\omega_i\}_{i=1}^m$ is a vector of weights.

Proposition 8 *Given an element* $x \in I^m$, *if we consider the vector* $\{\omega_i\}_{i=1}^m$ *associated to* x *by the expressions introduced in Definition 5, we have that*

$$\omega_i \ge 0, \forall i \in \{1, 2, \ldots, m\} \quad \text{and} \quad \sum_{i=1}^{m} \omega_i = 1.$$

Proof. By definition, it is immediate that $\omega_i \ge 0, \forall i \in \{1, 2, \ldots, m\}$ and

$$\sum_{i=1}^{m} \omega_i = w^*(1) - w^*(0) = 1 - 0 = 1. \blacksquare$$

3 The group decision making problem applied to the Risk Assessment Problem

In a group decision making problem we have a set of n alternatives $\mathcal{A} = \{a_1, \ldots, a_n\}$, $(n \geq 2)$ and a set of m experts $\mathcal{E} = \{e_1, \ldots, e_m\}$, $(m \geq 2)$. Each expert provides his preference on the set of alternatives and the goal of the group decision making problem is to look for the alternative (or set of alternatives) which is (are) most accepted by the experts. It must be noted that "different experts" could be understood as both "different people" or "different criteria".

The resolution of a group decision making problem, according with the principles established in [5], is developed in the following two steps:

(1) *Making the information uniform.* Each preference ordering of each expert is transformed into a fuzzy preference relation form.
(2) *Application of a selection process.* The most accepted alternative by our experts must be selected.
In addition, the selection process is also applied in two steps:
   (a) *Aggregation phase.* A consensus fuzzy preference relation is obtained using an aggregation operator.
   (b) *Exploitation phase.* The most accepted alternative from the consensus fuzzy preference relation is selected.

In the aggregation phase the use of linguistic quantifiers (for example "as many as possible") would be useful to represent the concept of fuzzy majority and they would allow us to build this collective fuzzy preference relation.

3.1 Making the information uniform

In the Risk Assessment Problem, for each alternative a_i, with $i = 1, 2, \ldots, n$, we have associated a vector with the utility values given by the m experts about it: $v_i = (v_i^1, \ldots, v_i^m)$. These values v_i^k are real numbers. However, the assessment of these utility values could be a bit uncertain in the Risk Assessment Problem and working with these utilities could be quite strict. Therefore, transform utilities into preference relations could provide our method with more flexibility.

Our goal is to construct m matrices, $P_1, \ldots, P_m$, representing the fuzzy preference relations associated to each expert in the set $\{e_1, \ldots, e_m\}$. Therefore, the problem is to look for a transformation function h allowing us to obtain the preference value (p_{ij}^k) measuring the degree of preference of e_i over e_j, for an expert e_k, depending only on the values of v_i^k and v_j^k:

$$p_{ij}^k = h(v_i^k, v_j^k), \; i, j \in \{1, 2, \ldots, n\} \text{ with } i \neq j \text{ and } k \in \{1, 2, \ldots, m\}.$$

Intuitively, the greater is the utility value given by the expert to the first alternative (v_i^k), the greater the preference relation should be. Analogously, the greater our second utility value v_j^k is, the lower the preference relation should be. Therefore, the transformation function h must be increasing in its first argument and decreasing in the second one.

Along this paper, we will consider the following transformation function:

$$p_{ij}^k = h(v_i^k, v_j^k) = \frac{\left(v_i^k\right)^r}{\left(v_i^k\right)^r + \left(v_j^k\right)^r}, \; i \neq j, r > 0. \tag{1}$$

It should be remarked that values of $r > 1$ benefit high utility values guiding us to a $\{0, 1\}$-value preference relation when $r \to +\infty$, and values of $r < 1$ decrease the differences between the risk values guiding us to a 0.5 single valued preference relation when $r \to 0$. The transformation function defined above is most common. However, other examples of transformation functions may be found in [5].

It is important to remark that experts can show their preference using utility values, fuzzy numbers or preference relations. In fact, they do not have to present them in the same way. These utilities represent a particular case used in our proposal to model a Risk Assessment Problem in order to obtain a preference relation which is the real initial point in a group decision making problem.

3.2 Application of a selection process: Aggregation phase

At this moment, we have m fuzzy preference relations, each of them expressing the preference of the respective expert. Thus, next step should be the "consensus" of the fuzzy preference relations.

Therefore, in this section it is explained how to aggregate the different fuzzy preference relations related with the different experts. In this kind of problems the OWA operators are often used and these operators are also related to the concept of fuzzy quantifiers. For example, the fuzzy quantifier "as many as possible" could be a perfect solution to the traditional group decision making problem based on looking for a "consensus" of different experts. However, in some cases (for instance in the application of Section 4), we are not just interested in a "consensus" of the different experts. Sometimes, the most important characteristic is to be "drastic" in at least one of the different objectives. That is the reason why the weight vector $w^{\text{OWA}} = (w_1^{\text{OWA}}, \ldots, w_m^{\text{OWA}})$ associated with the OWA operator must have its components ordered in a decreasing way, i.e.

$$w_1^{\text{OWA}} \geq w_2^{\text{OWA}} \geq \ldots \geq w_{n-1}^{\text{OWA}} \geq w_n^{\text{OWA}}.$$

When using an OWA operator, not all the different experts have the same degree of importance. In the traditional group decision making problem, assigning more importance to an expert over another could have a lack of sense or could derive in hierarchic problems. However, in other cases, it is totally necessary to assign an importance to each expert. Therefore, a WM operator could be considered to model this problem.

Thus, WOWA operator will be considered, as they combine these two points of view according to

$$p_{ij} = f_{\text{WOWA}}\left(p_{ij}^1, \ldots, p_{ij}^m\right),$$

if we have m fuzzy preference relations $P_1, \ldots, P_m$ associated to the m experts and the objective is to obtain a fuzzy preference relation P unifying these previous relations. Each element of the new preference relation, p_{ij}, represents the preference of alternative a_i over alternative a_j. Moreover, $f_{\text{WOWA}}(\cdot)$ is the WOWA operator with a weight vector w^{OWA} associated to the OWA part and a weight vector w^{WM} associated to the WM part.

3.3 Application of a selection process: Exploitation phase

Finally, once the final fuzzy preference relation is obtained, the different criteria are analysed in order to exploit the information.

In general, the previous steps of the method do not make the final fuzzy preference relation to be acyclic. For example, if we consider the risk vectors $(1,2,3,5)$, $(1,3,5,2)$ and $(1,5,2,3)$ with the weight vector $(1/4, 1/4, 1/4, 1/4)$ used in both parts of the WOWA operator, then the application of the above described steps lead us to the presence of cycles in P. Note that if the order of the first two steps is inverted ("making the information uniform" and "aggregation phase") then the method does not produce cycles (in fact, in this case it is a weak transitive method). However, we are not interested in changing this order, because the procedure obtained does not study the interaction between the different alternatives and some flexibility is lost.

Therefore, a non-transitive method must be used in this part of the analysis.

The weighted voting method ([22]) could direct to a more flexible study of the alternatives, where the importance of each event does not reside in "being desirable over the most events" but in "being the most desirable over the other events".

A natural extension of the weighted voting method is the main goal of this section. This extended method defines a parameter α allowing to model the importance of "being desirable" or "not being preferred" among the whole alternative set. This parameter represents the degree of optimism, as we will see later.

The algorithm proposed is the following.

Algorithm 1 Extended Weighted Voting Method (EWVM)

- ***Input:***
  · *A fuzzy preference relation P over a set of alternatives $\mathcal{A} = \{a_1, \ldots, a_n\}$*
  · *Fix the parameter α*
  · *Choose an aggregation operator Agg*
- ***Output:*** *A family of non-empty sets $\mathcal{K}_i$*

1. *Normalize P*
2. *Separate the "positive preference" (P^+) and the "negative preference" (P^-) via*
   $p_{ij}^+ = max\{0, p_{ij} - 0.5\}$
   $p_{ij}^- = min\{0, p_{ij} - 0.5\}$
3. $P^\alpha = \alpha \cdot P^+ + (1 - \alpha) \cdot P^-$
4. $\mathcal{B}_0 = \mathcal{A}$ *and* $k = 0$
5. *While* $\mathcal{B}_k \neq \emptyset$
   For each $a_i \in \mathcal{B}_k$
   $\mathbb{M}_i = \{l \in \{1, \ldots, n\} \ / \ a_l \in \mathcal{B}_k \text{ and } l \neq i\}$
   $I(a_i) = Agg\left(p_{i\mathbb{M}_i}^\alpha\right)$
   $\mathcal{K}_{k+1} = a_{argmax_i\{I(a_i)\}}$
   $\mathcal{B}_{k+1} = \mathcal{B}_k \backslash \mathcal{K}_{k+1}$
   $k = k + 1$
   end

Due to the handling of matrices in the aggregation phase, these matrices could derive on fuzzy preferences which not fulfil the normalization requirement. Thus, Step 1 is needed in order to re-obtain a fuzzy preference relation, otherwise, p_{ij}^+ and p_{ij}^- have no sense. Some examples of normalization functions may be found in [5].

In Step 2, p_{ij}^+ and p_{ij}^- represent respectively the values above or below the preference indifference between the alternatives, i.e. preference indifference holds when $p_{ij} = 0.5$.

On the other hand, when assigning the alternative (or alternatives) with the highest index to the set $\mathcal{K}_k$, a tie could be achieved, so a deadlock rule could be established in order to obtain a linear order.

Remark 9 *The sets $\mathcal{K}_i$ are disjoint sets satisfying $\bigcup\limits_i \mathcal{K}_i = \mathcal{A}$ for construction.*

A total order can be introduced using Algorithm 1, which is defined as follows.

Definition 10 *For any set of alternatives $\mathcal{A} = \{a_1, \ldots, a_n\}$ we can define the binary relations $\succ$, $\succeq$ and $\sim$ as follows:*

- $a_i \succ a_j \Leftrightarrow \exists \mathcal{K}_s, \mathcal{K}_t$ *such that* $a_i \in \mathcal{K}_s, a_j \in \mathcal{K}_t$ *and* $s < t$.
- $a_i \sim a_j \Leftrightarrow \exists \mathcal{K}_s$ *such that* $a_i \in \mathcal{K}_s, a_j \in \mathcal{K}_s$.
- $a_i \succeq a_j \Leftrightarrow a_i \succ a_j$ *or* $a_i \sim a_j$.

Proposition 11 *For any set of alternatives $\mathcal{A} = \{a_1, \ldots, a_n\}$, the relation defined in Definition 10 fulfil the following properties:*

(1) $\succeq$ and $\sim$ are reflexive.
(2) $\sim$ is symmetric and $\succeq$ is antisymmetric.
(3) $\succ$, $\succeq$ and $\sim$ are transitive.
(4) $\succeq$ is a total order.

Proof.

(1) By definition of $\{\mathcal{K}_i\}_i$, $\forall x \in \mathcal{A}$, $\exists \mathcal{K}_s$ such that $x \in \mathcal{K}_s$. So, $x \sim x$ and therefore $x \succeq x$.

(2) Let $x, y \in \mathcal{A}$ and $\mathcal{K}_i$ and $\mathcal{K}_j$ be the sets satisfying $x \in \mathcal{K}_i$ and $y \in \mathcal{K}_j$.
Therefore, $x \sim y \Rightarrow i = j \Rightarrow y \sim x$, i.e., $\sim$ is symmetric.
On the other hand if $x \succeq y$, then $i \leq j$. Analogously, if $y \succeq x$, then $j \leq i$. So, if $x \succeq y$ and $y \succeq x$, then $i = j$ and therefore $x \sim y$, i.e., $\succeq$ is antisymmetric.

(3) Let $x, y, z \in \mathcal{A}$ and $\mathcal{K}_i, \mathcal{K}_j, \mathcal{K}_k$ the sets satisfying $x \in \mathcal{K}_i$, $y \in \mathcal{K}_j$ and $z \in \mathcal{K}_k$.
If $x \sim y$ and $y \sim z$, then $i = j$ and $j = k \Longrightarrow i = k$ and $x \sim y$, i.e., $\sim$ is transitive.
If $x \succ y$ and $y \succ z$, then $i < j$ and $j < k \Longrightarrow i < k$ and $x \succ y$, i.e., $\succ$ is transitive.
If $x \succeq y$ and $y \succeq z$, then $i \leq j$ and $j \leq k \Longrightarrow i \leq k$ and $x \succeq y$, i.e., $\succeq$ is transitive.

(4) Let $x, y \in \mathcal{A}$ and $\mathcal{K}_i$ and $\mathcal{K}_j$ be the sets satisfying $x \in \mathcal{K}_i$ and $y \in \mathcal{K}_j$.
There are three possibilities $i < j$, $i > j$ and $i = j$:
If $i < j$, then $x \succ y$.
If $i > j$, then $y \succ x$.
If $i = j$, then $x \sim y$. ∎

Thus, it is proven that the relation $\sim$ is an equivalence relation and the relation $\succeq$ is a total order.

Remark 12 *Note that for $\alpha \in [0,1]$:*

- *If $\alpha = 0.5$, EWVM coincides with the "weighted voting method".*
- *If $\alpha < 0.5$, the "least dominated" alternative is preferred.*
- *If $\alpha > 0.5$, the "most dominating" alternative is preferred.*

The meaning of "least dominated" and "most dominating" alternative is shown in Example 13.

Example 13 *Suppose four alternatives a_1, a_2, a_3 and a_4. The consensus of the experts says that the alternatives a_1 and a_2 are the two most important alternatives. In particular the alternative a_1 is clearly more preferred than the alternatives a_3 and a_4, but the alternative a_2 is slightly more preferred than the other three, as we can see in the consensus preference relation:*

$$P = \begin{pmatrix} 0.5 & 0.45 & 1 & 1 \\ 0.55 & 0.5 & 0.6 & 0.6 \\ 0 & 0.4 & 0.5 & 0.5 \\ 0 & 0.4 & 0.5 & 0.5 \end{pmatrix}.$$

Then, is a_1 more important than a_2? It depends on the degree of optimism we want to assume. We are going to analyse the two extreme cases of $\alpha = 0$ and $\alpha = 1$.

If $\alpha = 0$, as $P^0 = P^-$, the preference is given to the "least dominated" alternative, obtaining the following indexes (considering the aggregation operator arithmetic mean):

$$I(a_1) = -0.0167, \qquad I(a_2) = 0, \qquad I(a_3) = I(a_4) = -0.2.$$

Therefore, we will classify as the most important alternative a_2.

On the other hand, if the preference is given to the "most dominating" alternative (with $\alpha = 1$), as $P^1 = P^+$, the indexes are

$$I(a_1) = 0.3333, \qquad I(a_2) = 0.0833, \qquad I(a_3) = I(a_4) = 0.$$

Consequently, the most preferred alternative in this case is a_1.

3.4 Behaviour of EWVM depending on α

Previously, the extended weighted voting method was introduced and, as it was detailed, the algorithm depends on parameter α. In this section, it is studied more in depth the performance of the algorithm according to the values of α. The objective is to check how the method depends on the parameter α and how variations in α do not drastically affect the final result if there is a significant order between the alternatives.

Therefore, this experiment is divided in two parts. In the first part random matrices are used to study the dependence of EWVM on the parameter and then, in the second part, matrices with a sense of transitivity between their respective alternatives are used in order to study the suitable behaviour of EWVM.

Firstly, a hundred of 100×100 matrices have been randomly generated. These matrices are supposed to be the fuzzy preference relation between 100 certain alternatives, so all their components are random values between 0 and 1 satisfying $p_{ij} + p_{ji} = 1 \; \forall i, j$.

Let us study how the output of the method is influenced by the value of parameter α. Therefore, several values of α have been fixed: $\alpha_1 = 0$, $\alpha_2 = 0.25$, $\alpha_3 = 0.5$, $\alpha_4 = 0.75$ and $\alpha_5 = 1$.

After applying the extended weighted voting method to each matrix with each level of α, 500 orders of our 100 alternatives are obtained. Spearman and Kendall coefficients (see [17]) can be used to measure statistical dependence between these ordinal variables.

The following matrices represent Spearman (ρ) and Kendall (τ) coefficients for the 5 different levels of α. Each element of ρ and τ, represents the corresponding averaged (over the values obtained for each one of the 100 initial matrices) coefficient between the orders induced by α_i and α_j:

$$\rho = \begin{pmatrix} 1.0000 & 0.7894 & 0.6356 & 0.4630 & 0.3305 \\ 0.7894 & 1.0000 & 0.7990 & 0.6422 & 0.5165 \\ 0.6356 & 0.7990 & 1.0000 & 0.8103 & 0.7059 \\ 0.4630 & 0.6422 & 0.8103 & 1.0000 & 0.8531 \\ 0.3305 & 0.5165 & 0.7059 & 0.8531 & 1.0000 \end{pmatrix},$$

$$\tau = \begin{pmatrix} 1.0000 & 0.6230 & 0.4721 & 0.3306 & 0.2321 \\ 0.6230 & 1.0000 & 0.6323 & 0.4770 & 0.3716 \\ 0.4721 & 0.6323 & 1.0000 & 0.6464 & 0.5332 \\ 0.3306 & 0.4770 & 0.6464 & 1.0000 & 0.6945 \\ 0.2321 & 0.3716 & 0.5332 & 0.6945 & 1.0000 \end{pmatrix}.$$

Note that low values of Spearman and Kendall coefficients are associated to the farthest values of α. Therefore, the more different the values of α, the more different the orders induced. So, EWVM leads us to different orders.

Other question arising is related to the behaviour of EWVM when there is some transitivity between the preferences of the alternatives. To check it, a hundred of 100×100 matrices have been generated, but not in a random way. In order to obtain each 100×100 matrix, some utility values have been randomly generated for every alternative and then the matrices have been constructed using these values. As there is a "sense of transitivity" in the way we have constructed these matrices, the behaviour of the alternatives would not be as chaotic as before and the most obvious orders between our alternatives would be kept. However, it must be remarked that we are not constructing strictly transitive matrices, but matrices where the behaviour is "more transitive" than before.

Finally, EWVM was applied to obtain different preference relation matrices.

The results obtained in this case are:

$$\rho = \begin{pmatrix} 1.0000 & 0.9976 & 0.9972 & 0.9971 & 0.9970 \\ 0.9976 & 1.0000 & 0.9999 & 0.9998 & 0.9998 \\ 0.9972 & 0.9999 & 1.0000 & 1.0000 & 0.9999 \\ 0.9971 & 0.9998 & 1.0000 & 1.0000 & 1.0000 \\ 0.9970 & 0.9998 & 0.9999 & 1.0000 & 1.0000 \end{pmatrix},$$

$$\tau = \begin{pmatrix} 1.0000 & 0.9690 & 0.9663 & 0.9651 & 0.9645 \\ 0.9690 & 1.0000 & 0.9968 & 0.9955 & 0.9948 \\ 0.9663 & 0.9968 & 1.0000 & 0.9987 & 0.9980 \\ 0.9651 & 0.9955 & 0.9987 & 1.0000 & 0.9993 \\ 0.9645 & 0.9948 & 0.9980 & 0.9993 & 1.0000 \end{pmatrix}.$$

In this case, the values of the coefficients are higher, therefore there is a stronger relation between the orders induced by the different values of α.

Therefore, as shown in the first part of the experiment, EWVM absolutely depends on the parameter α. However, we can observe that, as shown in the second part of the experiment, under certain conditions of transitivity this method leads us to similar orders where the differences are just in the few alternatives more likely to move its position when the criteria is ranged from the "most dominating" to the "least dominated".

4 Application: Multifactorial Risk Assessment Evaluation

As we commented in the introduction, the developed method was considered as a solution to a real problem. In this section we will explain this problem and how the solution is obtained by means of the previous procedure.

In this particular framework, our purpose was to order different human errors (in this case they play the role of alternatives) in accordance with their importance for several criteria (which can be considered as experts). Any criterion will allow us to obtain a fuzzy preference relation associated to the human errors. The evaluation of the alternatives to obtain this relation is given by means of risk assessment matrices. These matrices are probably one of the most widespread tools for studying the risk evaluation problem. They are mainly used to determine the importance of a risk and whether the risk is sufficiently controlled or not.

A risk matrix has two dimensions. It represents how severe and likely an unwanted event is. The combination of probability and severity will give to any event a place on a risk matrix (there are some events that are more difficult, but well come to that later). This is a simple mechanism to ease the visibility of the most dangerous risks and assist in the decision making process.

Although many standard risk matrices exist in different contexts: US DoD, NASA, ISO (see [10,15,19]), individual projects and organizations may need to create their own risk assessment matrix or tailor an existing one. In this paper, it is considered the standard created for the FASyS project (which is the Spanish acronym for "Fábrica Absolutamente Segura y Saludable" that means "Absolutely Safe and Healthy Factory") [8], since it is the most frequently considered one in Spain for Transportation Systems.

In our study, the harm severity is categorized as: severe, significant, moderate, minor, negligible and none. On the other hand, the probability of harm occurring is categorized as: frequent, probable, occasional, remote and improbable. Therefore, each hazard level is associated with a risk value, which is represented, in this case, by a number. The aforementioned risk assessment matrix is shown in Table 1.

		SEVERITY					
		None	Negligible	Minor	Moderate	Significant	Severe
LIKELIHOOD	Frequent	8	16	24	40	64	104
	Probable	5	10	15	25	40	65
	Occasional	3	6	9	15	24	39
	Remote	2	4	6	10	16	26
	Improbable	1	2	3	5	8	13

Table 1
Risk assessment matrix.

As it can be seen from the table, the risk matrix has three areas:

- The high probability-high severity area (red) which indicates that an event categorized in this area needs to be solved immediately, because its consequences would be catastrophic.
- The low probability-low severity area (green) which indicates that the risk of an event is not high enough to be taken into account, or that it is sufficiently controlled. No action is usually taken for events in this area.
- The medium category (yellow) is located between these two areas. Any event that falls in this area needs to be monitored, but giving preference to the incidents in the red zone.

Even though the risk assessment matrix has a lot of drawbacks, it is still one of the standard tools used in most risk assessment problems and, if properly used, it can provide a reasonable solution.

Several organizations use the risk assessment matrix as an essential tool for classifying the different errors according to their importance. This matrix is usually based only on the economic impact for the company. However, nowadays, other important impacts can be derived from an error, such as reputation of the enterprise or consequences for people. Moreover, the environmental impact is a growing concern for most of the companies. Thus, these four concerns (assets, reputation, people and environment) could be analysed by experts in order to classify the errors. According to that, several different risk assessment matrices should be managed at the same time (one for each way of consequence) and we would like to obtain an order for the importance of the possible errors.

Let us suppose six possible errors with the estimation about their likelihood and severity shown in Table 2 (for simplicity the likelihood is encoded as 1-improbable, 2-remote, 3-occasional, 4-probable and 5-frequent and analogously the severity as: 1-none, 2-negligible, 3-minor, 4-moderate, 5-significant and 6-severe).

		Reputation	Assets	People	Environment
Event code	Likelihood	Severity			
E1	5	2	2	2	1
E2	2	6	1	4	6
E3	1	3	2	4	1
E4	2	4	6	5	1
E5	2	6	4	5	2
E6	5	2	3	3	1

Table 2
Estimation of likelihood and severity of each event.

Therefore, the associated risk vectors for each event are obtained from the risk assessment matrix, as we can see in Table 3.

Event code	Risk vector
E1	(16,16,16,8)
E2	(26,2,10,26)
E3	(3,2,5,1)
E4	(10,26,16,2)
E5	(26,10,16,4)
E6	(16,24,24,8)

Table 3
Risk vectors obtained for each event.

The main drawback of this (necessary) proposal is that there exists no total order when working in $\mathbb{R}^4$. If we have an event with a risk vector of $(10, 26, 16, 2)$, it is clear that it is more important than an event with risk values $(3, 2, 5, 1)$. But, what happens with an incident whose risk values are $(16, 24, 24, 8)$? It is more important in accordance with the second criterion and less important in accordance with the other three criteria.

This situation led us to work with a partial order in $\mathbb{R}^n$, while needing to obtain a total order for the set of alternatives. In the previous section a group decision making problem was considered. If the experts would be the different criteria to study, we could consider this method to solve this problem. Therefore, in this case $m = 4$.

The starting point here are the risk vectors v_i^k shown in Table 3 for the 6 events. Therefore, proceeding with the first step of a group decision making problem, the information is made uniform, i.e. the four fuzzy preference relations P^1

(reputation), P^2 (assets), P^3 (people) and P^4 (environment) are constructed. These preference relations are computed using the Equation 1 with $r = 1$.

For example, $p^1_{12} = \dfrac{16}{16+26} \approx 0.3810$ represents the degree of preference of event $E1$ over event $E2$, in relation with the damages to the reputation of the enterprise. The four fuzzy preference relations are shown below

$$P^1 = \begin{pmatrix} 0.5000 & 0.3810 & 0.8421 & 0.6154 & 0.3810 & 0.5000 \\ 0.6190 & 0.5000 & 0.8966 & 0.7222 & 0.5000 & 0.6190 \\ 0.1579 & 0.1034 & 0.5000 & 0.2308 & 0.1034 & 0.1579 \\ 0.3846 & 0.2778 & 0.7692 & 0.5000 & 0.2778 & 0.3846 \\ 0.6190 & 0.5000 & 0.8966 & 0.7222 & 0.5000 & 0.6190 \\ 0.5000 & 0.3810 & 0.8421 & 0.6154 & 0.3810 & 0.5000 \end{pmatrix},$$

$$P^2 = \begin{pmatrix} 0.5000 & 0.8889 & 0.8889 & 0.3810 & 0.6154 & 0.4000 \\ 0.1111 & 0.5000 & 0.5000 & 0.0714 & 0.1667 & 0.0769 \\ 0.1111 & 0.5000 & 0.5000 & 0.0714 & 0.1667 & 0.0769 \\ 0.6190 & 0.9286 & 0.9286 & 0.5000 & 0.7222 & 0.5200 \\ 0.3846 & 0.8333 & 0.8333 & 0.2778 & 0.5000 & 0.2941 \\ 0.6000 & 0.9231 & 0.9231 & 0.4800 & 0.7059 & 0.5000 \end{pmatrix},$$

$$P^3 = \begin{pmatrix} 0.5000 & 0.6154 & 0.7619 & 0.5000 & 0.5000 & 0.4000 \\ 0.3846 & 0.5000 & 0.6667 & 0.3846 & 0.3846 & 0.2941 \\ 0.2381 & 0.3333 & 0.5000 & 0.2381 & 0.2381 & 0.1724 \\ 0.5000 & 0.6154 & 0.7619 & 0.5000 & 0.5000 & 0.4000 \\ 0.5000 & 0.6154 & 0.7619 & 0.5000 & 0.5000 & 0.4000 \\ 0.6000 & 0.7059 & 0.8276 & 0.6000 & 0.6000 & 0.5000 \end{pmatrix},$$

$$P^4 = \begin{pmatrix} 0.5000 & 0.2353 & 0.8889 & 0.8000 & 0.6667 & 0.5000 \\ 0.7647 & 0.5000 & 0.9630 & 0.9286 & 0.8667 & 0.7647 \\ 0.1111 & 0.0370 & 0.5000 & 0.3333 & 0.2000 & 0.1111 \\ 0.2000 & 0.0714 & 0.6667 & 0.5000 & 0.3333 & 0.2000 \\ 0.3333 & 0.1333 & 0.8000 & 0.6667 & 0.5000 & 0.3333 \\ 0.5000 & 0.2353 & 0.8889 & 0.8000 & 0.6667 & 0.5000 \end{pmatrix}.$$

Next step is the aggregation of the four matrices to obtain a final fuzzy preference relation P. Let us consider the WOWA operator with a weight vector $w^{\mathrm{WM}} = (1/4, 1/4, 1/3, 1/6)$ associated to the WM part and a weight vector $w^{\mathrm{OWA}} = (1/2, 1/3, 1/9, 1/18)$ associated to the OWA part. Finally, the linear interpolation was considered as w^* in this example.

In Example 7, we calculated the element

$$p_{12} = f_{\mathrm{WOWA}}\,(0.3810, 0.8889, 0.6154, 0.2353) = 0.7164.$$

Using the same procedure for each p_{ij}, the following consensus fuzzy preference relation is obtained:

$$P = \begin{pmatrix} 0.5 & 0.7164 & 0.8685 & 0.6383 & 0.5938 & 0.4722 \\ 0.5873 & 0.5 & 0.8456 & 0.6798 & 0.5781 & 0.5653 \\ 0.2008 & 0.3844 & 0.5 & 0.2598 & 0.2142 & 0.1588 \\ 0.5377 & 0.7206 & 0.8442 & 0.5 & 0.5864 & 0.4512 \\ 0.5427 & 0.6958 & 0.8559 & 0.6358 & 0.5 & 0.4987 \\ 0.5870 & 0.7670 & 0.8916 & 0.6660 & 0.6556 & 0.5 \end{pmatrix}.$$

Now, we will proceed with the extended weighted voting method. Firstly, the final fuzzy preference relation is normalized dividing the values of p_{ij} and p_{ji} by their sum.

$$P' = \begin{pmatrix} 0.5 & 0.5495 & 0.8122 & 0.5427 & 0.5225 & 0.4458 \\ 0.4505 & 0.5 & 0.6875 & 0.4855 & 0.4538 & 0.4243 \\ 0.1878 & 0.3125 & 0.5 & 0.2353 & 0.2002 & 0.1512 \\ 0.4573 & 0.5146 & 0.7647 & 0.5 & 0.4798 & 0.4039 \\ 0.4775 & 0.5462 & 0.7998 & 0.5202 & 0.5 & 0.4320 \\ 0.5542 & 0.5757 & 0.8488 & 0.5961 & 0.5680 & 0.5 \end{pmatrix}.$$

Then, the "positive preference" (P^+) and the "negative preference" (P^-) are separated.

$$P^{+} = \begin{pmatrix} 0 & 0.0495 & 0.3122 & 0.0427 & 0.0225 & 0.0000 \\ 0.0000 & 0 & 0.1875 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0146 & 0.2647 & 0 & 0.0000 & 0.0000 \\ 0.0000 & 0.0462 & 0.2998 & 0.0202 & 0 & 0.0000 \\ 0.0542 & 0.0757 & 0.3488 & 0.0961 & 0.0680 & 0 \end{pmatrix},$$

$$P^{-} = \begin{pmatrix} 0 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & -0.0542 \\ -0.0495 & 0 & 0.0000 & -0.0145 & -0.0462 & -0.0757 \\ -0.3122 & -0.1875 & 0 & -0.2647 & -0.2998 & -0.3488 \\ -0.0427 & 0.0000 & 0.0000 & 0 & -0.0202 & -0.0961 \\ -0.0225 & 0.0000 & 0.0000 & 0.0000 & 0 & -0.0680 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0 \end{pmatrix}.$$

Using $\alpha = 0.25$ we will obtain this weighted matrix:

$$P^{\alpha} = \begin{pmatrix} 0 & 0.0124 & 0.0781 & 0.0107 & 0.0056 & -0.0407 \\ -0.0371 & 0 & 0.0469 & -0.0109 & -0.0347 & -0.0568 \\ -0.2342 & -0.1406 & 0 & -0.1985 & -0.2249 & -0.2616 \\ -0.0320 & 0.0037 & 0.0662 & 0 & -0.0152 & -0.0721 \\ -0.0169 & 0.0116 & 0.0750 & 0.0051 & 0 & -0.0510 \\ 0.0136 & 0.0189 & 0.0872 & 0.0240 & 0.0170 & 0 \end{pmatrix}.$$

Choosing the arithmetic mean as aggregation operator, the following indexes are obtained:

$$\begin{aligned} &I(E1) = 0.0132, && I(E2) = -0.0185, && I(E3) = -0.2120, \\ &I(E4) = -0.0099, && I(E5) = 0.0047, && I(E6) = 0.0321. \end{aligned}$$

Then, the event $E6$ is classified at the first position (because it has the highest index) and we will remove it obtaining the following weighted matrix:

$$P^{\alpha} = \begin{pmatrix} 0 & 0.0124 & 0.0781 & 0.0107 & 0.0056 & \cancel{-0.0407} \\ -0.0371 & 0 & 0.0469 & -0.0109 & -0.0347 & \cancel{-0.0568} \\ -0.2342 & -0.1406 & 0 & -0.1985 & -0.2249 & \cancel{-0.2616} \\ -0.0320 & 0.0037 & 0.0662 & 0 & -0.0152 & \cancel{-0.0721} \\ -0.0169 & 0.0116 & 0.0750 & 0.0051 & 0 & \cancel{-0.0510} \\ \cancel{0.0136} & \cancel{0.0189} & \cancel{0.0872} & \cancel{0.0240} & \cancel{0.0170} & \cancel{0} \end{pmatrix}.$$

At the next iteration, the indexes for the current 5 alternatives are:

$$I(E1) = 0.0267, \quad I(E2) = -0.0089, \quad I(E3) = -0.1995,$$
$$I(E4) = 0.0057, \quad I(E5) = 0.0187.$$

Therefore, the event $E1$ is classified at the second position.

Following with the algorithm until all events are classified, the obtained order is:

$$E6 > E1 > E5 > E4 > E2 > E3.$$

Hence, the company must work the most in solving the event $E6$ in order to decrease the risk of accident due to human error.

5 Conclusions and future research

In this work we have presented a procedure for aggregating preference relations representing the opinion of different experts: the Extended Weighted Voting Method. The introduction of this method was a key contribution in this paper and it will allow us to obtain different admissible orders. These obtained orders could be modelled using a parameter α, which measures the degree of optimism acquired. A deep study of the influence of this parameter α was also shown. Finally, this procedure was used for aggregating risk assessment matrices associated with different criteria. We have particularized on the study of 4 criteria: reputation, assets, people and environment. We have considered this Human Reliability problem as a group decision making problem, and we have also used the WOWA operator to search the most accepted alternative, but considering at the same time the influence of the "reliability" of our experts.

In the future we will intend to go deeper into the analysis of this problem. In particular, we would like to consider different methods in the exploitation phase and to introduce linguistic labels in the definition of the risk assessment matrix. This last project could derive on the definition of a totally new risk assessment matrix and in the study of the joint of linguistic labels. Moreover, we would like to apply our study to real problems and compare our results with the results obtained with the techniques used nowadays.

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